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# SPLINE ANALYSIS OF THE MANDIBLE IN HUMAN SUBJECTS WITH CLASS III MALOCCLUSION

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Summary—This study determines deformations that contribute to a Class III mandibular morphology, employing thin-plate spline (TPS) analysis. A total of 133 lateral cephalographs of prepubertal children of European-American descent with either a Class I molar occlusion or a Class III malocclusion were compared. The cephalographs were traced and checked, and eight homologous landmarks on the mandible were identified and digitized. The datasets were scaled to an equivalent size and subjected to statistical analyses. These tests indicated significant differences between average Class I and Class III mandibular morphologies. When the sample was subdivided into seven age and sex-matched groups statistical differences were maintained for each group. TPS analysis indicated that both affine (uniform) and non-affine transformations contribute towards the total spline, and towards the average mandibular morphology at each age group. For non-affine transformations, partial warp 5 had the highest magnitude, indicating large-scale deformations of the mandibular configuration between articulare and pogonion. In contrast, partial warp 1 indicated localized shape changes in the mandibular symphyseal region. It is concluded that large spatial-scale deformations affect the body of the mandible, in combination with localized distortions further anteriorly. These deformations may represent a developmental elongation of the mandibular corpus antero-posteriorly that, allied with symphyseal changes, leads to the appearance of a Class III prognathic mandibular profile. © 1997 Elsevier Science Ltd

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### INTRODUCTION

The incidence of Class III malocclusion accounts for under 4% of all malocclusions (for a review, see Van Vuuren, 1991). Mandibular prognathism is a common clinical finding in this particular class of malocclusions. For example, Nakasima et al. (1986) found a familial tendency of mandibular prognathism in skeletal Class III cases. Similarly, Mackay et al. (1992) identified five Class III subgroups, all of which exhibited mandibular prognathism. The aetiology of Class III malocclusion, however, remains unresolved. Kerr and Ten Have (1988) suggest that overclosure and anterior displacement of the mandible may play a part, but it is unclear whether Class III malocclusion characterized by jaw discrepancy is caused by variations in mandibular position, mandibular size or a combination of the two (Kerr et al., 1994).

There is a dearth of morphometric studies that assess mandibular morphology without resorting to

conventional cephalometric analysis. Kerr et al. (1994), using the method of superimposition, determined mean mandibular plots centred on the sellanasion and gonion-gnathion planes and suggested that the position of Class III mandibles is more anterior and rotated forward compared to other occlusal groups. But Moyers and Bookstein (1979) have noted the inappropriateness of cephalometry. In this regard, Lavelle (1984) bypassed intrinsic cephalometric deficiencies. Using medial axis transformation, he reported constancy of mandibular form regardless of occlusal classification, and while demonstrating that mandibular shape appears to be less variable than its size, he was unable to account for skeletal jaw relations typified by the Class III malocclusion.

Thin-plate spline is a more rigorous quantitative analysis of the spatial organisation of shape change (Bookstein, 1989). It expresses the differences between two configurations as a continuous deformation, using regression functions in which homologous landmarks are matched exactly between the two forms, explicitly to minimize the bending energy (Richtsmeier *et al.*, 1992). Bending energy can be

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Abbreviations: Cephalometric landmarks are explained in the text and Fig. 1.

thought of as the energy that would be required to bend an infinitely thin metal plate over one set of landmarks so that the height over each landmark is equal to the coordinates of the corresponding point in the other form. The specific interpolation function is a mathematical expression for the deformation of theoretically idealized thin metal plates (Bookstein, 1989). These properties enable the construction of transformation grids such as those associated with D'Arcy Thompson (1917).

The deformation can be decomposed into a series of geometric components: affine and non-affine changes. The affine transformations correspond to changes due to size difference, rotation and uniform shape change. The non-affine changes correspond to non-uniform or local deformation; each component is a weighted combination of landmark displacements. Moreover, if the objects are two-dimensional, these non-affine changes can be inspected visually for biological interpretations (Richtsmeier et al., 1992). The non-affine changes can be decomposed further into a series of more localized components represented by partial warps, components corresponding to deformations at different geometric scales. The number of these partial warps is three fewer than the number of landmarks (Bookstein, 1989, 1991; Swiderski, 1993; Singh et al., 1997a). The most global components are combinations that represent transformations affecting the entire form. The other components represent a series of transformations affecting progressively smaller regions, down to highly localized changes affecting the immediate vicinity of a few closely spaced landmarks (Swiderski, 1993).

The mode of each partial warp is determined by the configuration of landmarks in the starting form and represents the mode of relative landmark displacements for shape changes at that scale of localization. These modal forms are called the principal warps; eigenvectors of the bending energy matrix (Bookstein, 1989). Therefore, the contribution of change at the scale of a particular principal warp to the realized landmark displacements in the x, yplane is expressed as a vector, the partial warp, i.e. the multipliers of the eigenvectors (the thin-plate spline-weighted sums of the principal warps) are the partial warps. Eigenvalues are interpreted at an inverse index to the scale of the corresponding principal warp; more energy is required to bend the thin plate between closely spaced landmarks than between remote landmarks. Therefore, eigenvalues are inversely related to the spatial scale of the feature; high eigenvalues are associated with highly localized features and high bending energy. As magnitude is a measure of how important a principal warp is for fitting the second form, partial warps with large magnitudes are interpreted as making the most difference (Zelditch et al., 1992, 1993). For these partial warps, the bending energy is the product of the magnitude of the warp and its eigenvalue (Bookstein 1989, 1991). Thus, principal warps are geometric terms in which morphological differences can be described, and partial warps are the values assigned to them (Swiderski, 1993).

Singh et al. (1997a) previously employed thinplate spline analysis in an examination of the role of the cranial base in the aetiology of Class III malocclusion. Therefore, our overall aim of the present study is to provide evidence for the theory that antero-posterior elongation of the mandible, presumably due to condylar proliferation, is associated with the development of Class III malocclusions. Therefore, our specific objectives were to undertake a thin-plate spline analysis of mandibular morphology, to localize primary morphological sites, and to assess their contibutions to the appearance of a Class III facial profile. We tested the hypothesis that a specific pattern of mandibular transformation is associated with Class III malocclusion and that visualization of those deformations is feasible using thin-plate splines.

### MATERIALS AND METHODS

The total sample included an approximately equal number of male and female patients of the same ethnic group, with a negative history of airway problems, and no obvious vertical skeletal problems. Pretreatment lateral cephalographs (133 in total) of patients with a Class III molar malocclusion (Guyer *et al.*, 1986) between the ages 5 to 11 years or an untreated Class I molar occlusion were employed. The group with Class III maloccusion consisted of 73 patients; the group with normal occlusion consisted of 60 individuals. Each occlusal group (Class I or Class III) was also subdivided into seven age subgroups (approx. 10 children per subgroup, with approximately equal numbers of males and females in each subgroup).

The magnification of each film was standardized to an 8% enlargement factor. The chronological age was assumed to match developmental age in this study as carpal ages were unavailable. Each lateral cephalograph was traced on frosted acetate film (0.03 in thick) and checked by one investigator (GDS). Landmark coordinates from cephalographs taped to a light box of uniform brightness were digitized using appropriate software and a digitizing tablet (Numonics Inc., Montgomeryville, PA). Eight homologous mandibular landmarks were digitized [Fig. 1(a)]. Any landmarks that demonstrated a discrepancy of >1% for each x, y coordinate on duplicate digitization were deemed to be identified unreliably and were excluded from the final analyses

For the Class III and Class I groups, the scaled, mean mandibular morphology was derived, employing Procrustes analysis (Goodall, 1991; Singh *et al.*, 1997b). These mean forms were tested for significance under the assumption of equivalence of var-



Fig. 1. (a) Mandibular homologous landmarks employed. Co, condylion (superiormost point on mandibular condyle); Ar, articulare (intersection of condylar head and posterior cranial base); Go, gonion (midpoint at angle of mandible); M, menton (inferiormost point on mandibular symphysis); Gn, gnathion (most anteroinferior point on mandibular symphysis); Po, pogonion (anteriormost point on mandibular symphysis); B, suprementale (point B: deepest point on mandibular alveolus); Id, infradentale (most anterosuperior point on the mandibular alveolus). (b) Illustration of overall mandibular configuration derived from the mandibular landmarks employed superimposed upon the tracing of a cephalograph of a Class III individual.

iance. These mean forms were also subjected to Euclidean distance-matrix analysis (Lele and Cole, 1996) to corroborate the Procrustes analysis, as well as bivariate, pairwise *t*-tests on selected mandibular variables. These statistical tests indicated that significant differences were demonstrable when the mean Class I mandibular configuration was compared to the mean Class III configuration. Therefore, the mean Class I configuration was also derived for each of the seven age subgroups, and compared to the corresponding mean Class III configuration.

For the overall analysis of geometric transformation from a normal to a Class III morphology, the scaled, mean mandibular configuration for normal and Class III forms was employed [Fig. 1(b)]. Similarly, at each age subgroup, the mean Class I mandibular morphology was transformed into the corresponding mean Class III configuration computed for that specific age. All mandibular configurations of the mean forms were subjected to thinplate spline analysis using appropriate software (Rohlf, 1996a). Each total spline was decomposed into an affine and non-affine component. The affine transformation describes the uniform component of change and, requiring no 'geometrical work', has no bending energy. The non-affine component, however, can be decomposed further into partial warps, the magnitude representing the importance of the partial warp to the total fit. Therefore, grids depicting deformations of the mandible transforming a Class I configuration into a Class III morphology were interpreted on this basis, but only the overall comparison and those at ages 5, 7, 9 and 11 years are illustrated for the sake of brevity.

## RESULTS

Figure 2 shows the mean mandibular configuration of the Class I morphology derived from the 60 Class I individuals in untransformed space.

Figure 3(a) demonstrates the pattern of deformation (the total spline) of the mean Class I grid



Fig. 2. Mean mandibular configuration of the Class I morphology derived from 60 Class I individuals in untransformed space.



Fig. 3. (a) Total spline for overall pattern of deformation when transforming a Class I morphology into the Class III configuration. The transformation grid shows evidence of an anteroposterior elongation and anterior displacement at Ar. The overall pattern of deformation is very similar to that for the sub groups at age 5–11 years. (b) Partial warp 5: it has the highest magnitude, and indicates large spatialscale deformations affecting the mandibular configuration. The effect of the partial warp appears to be anteroposterior stretch between articulare and pogonion. (c) Partial warp 4: it has the lowest magnitude and acts at more closely located landmarks. Anterior displacement of articulare is one feature that appears to be associated with this partial warp 2: demonstrates a more localized deformation of the symphyseal region. (e) Partial warp 2: demonstrates deformation of the symphyseal region, particularly between menton and infradentale. (f) Partial warp 1: indicates localized shape changes in the mandibular symphyseal region.

when it is transformed into the mean morphology derived from 73 Class III individuals, using thinplate spline analysis. When transforming the Class I configuration into Class a III morphology, the transformation grid showed evidence of an anteroposterior elongation, most striking along the gonion-infradentale (Go-Id) axis. Anterior displacement at articulare (Ar) was accompanied by minor vertical compression between infradentalesupramentale (Id-B) and vertical elongation between supramentale-pogonion (B-Po). There was evidence of horizontal compression between menton and gnathion (M-Gn). When the total spline of the overall transformation was decomposed into partial warps, the pattern of deformation was confirmed. Partial warp 5 had the highest magnitude [Table 1; Fig. 3(b)], indicating large-scale deformations stretching the mandibular transformation grid between Ar-Po. In contrast, partial warp 4 had the lowest magnitude [Table 1; Fig. 3(c)], indicating more localized shape changes in the condylar region with anterior displacement at Ar. Partial warps 1–3 were of similar magnitude and demonstrated deformation of the symphyseal region posteriorly, superiorly and anteroinferiorly [Table 1; Fig. 3(d)–(f)].

Age (years)	Partial warp	Eigenvalue	Energy $\times 10^{-3}$	Magnitude ×10 <sup>-5</sup>
Overall	1	230.07	7.87	3.42
	2	70.70	2.55	3.60
	3	39.65	1.51	3.81
	4	23.81	0.76	3.19
	5	3.23	0.39	11.95
5	1	241.99	8.64	3.57
	2	65.10	5.99	9.20
	3	38 73	1.82	4 71
	4	21.55	0.72	3 32
	5	3.15	0.36	11.51
6	1	230.29	18.03	7 83
	2	87 47	2 15	7.65
	2	38 50	2.15	15.60
	3	30.39	0.02	12.00
	5	3.38	5.88 <b>1.70</b>	<b>50.27</b>
7		201.45	22.14	11.50
	1	201.45	23.16	11.50
	2	74.58	4.70	6.30
	3	38.46	2.78	7.22
	4	22.20	0.27	1.23
	5	3.20	0.17	5.24
8	1	268.02	2.09	0.78
	2	70.69	0.34	0.48
	3	44.18	0.39	0.89
	4	24.94	0.49	1.96
	5	3.20	0.08	2.35
9	1	231.16	10.54	4.56
	2	68.73	1.51	2.20
	3	39.24	1.86	4 74
	4	25.21	1.38	5.45
	5	3.24	0.45	13.97
10	1	229.05	4 14	1.81
	2	75.67	0.16	0.20
		39 79	0.81	2.04
	4	22.24	1 28	5 74
	5	3.35	0.56	16.58
11	1	222.88	10.51	1 72
	2	61.06	6 21	4.72
	2	36.20	0.21	0.82
	5 A	23.06	0.50	0.65
	7 5	23.00	0.29	1.24
	5	3.11	0.90	30./9

Table 1. Contribution of each partial warp towards the total spline

Note that when the overall normal (Class I occlusion) configuration is transformed to a mandibular prognathic configuration (Class III malocclusion) the transformation indicates that partial warp 5 has the highest magnitude, making the largest contribution to the change in morphology. This pattern is reflected also when the sample is subdivided into each age group (except at age 7 years) The pattern of deformation was also very similar at each of the seven subgroups at ages 5–11 years. On examining the 5-year-old mandibular transformation grid (Fig. 4), deformations in the attainment of the Class III configuration showed some anterior displacement at Ar with an anteroposterior stretch along the Go–Id axis. Vertical compression in the Id–B region was present in contrast to slight vertical elongation at landmarks between B and M.

For the 6-year-old transformation from a Class I to a Class III mandibular morphology, vertical stretch of the configuration was evident (not illustrated). An anterior displacement at Ar was accompanied by an anteroposterior stretch along the Go–Id axis. While vertical elongation between Id and M was also a feature, horizontal compression of symphyseal landmarks Po–M was evident.

Unlike the other age subgroups, for the 7-yearold mandibular transformation grid (Fig. 5), the Class III configuration was predominantly attained through vertical stretch. This is reflected in Table 1 that shows that partial warp 5 does not have the largest magnitude at this particular age. There was anterior displacement at Ar, however, with an anteroposterior stretch along the Go–Id axis. Vertical compression in the Id–B region was apparent but vertical elongation between B and M was a salient feature.

For the 8-year-old Class III transformation, horizontal stretch of the Class I mandibular configuration was the most evident feature (not illustrated). Minor anterior displacement at Ar and anteroposterior stretch along the Go–Id axis was present, but there was little evidence of deformation in the symphyseal region.

For the 9-year-old Class III mandibular transformation grid (Fig. 6), vertical stretch of the Class



Fig. 4. Total spline of the 5-year-old mandibular configuration. To transform a Class I geometry to a Class III configuration for this age group, anteroposterior stretch along the Go-ld axis, anterior displacement at Ar and symphyseal deformation appears to be necessary.



Fig. 5. Total spline of the 7-year-old configuration. To transform a Class I geometry to a Class III configuration, there is evidence of vertical stretch, anterior displacement at Ar, anteroposterior stretch along the Go–Id axis and symphyseal deformation at this age.

I configuration was evident. Minor anterior displacements at Ar with anteroposterior stretch along the Go–Id axis were also present. Vertical stretch in the Id–Gn region was seen but horizontal compression between Gn and M was also present.

For the 10-year-old transformation grid, anterior displacements at Ar with anteroposterior stretch along the Go–Id axis was required to achieve the Class III mandibular configuration (not illustrated). Some vertical stretch in the Id–Po region was present but it was accompanied by minor horizontal compression between Gn and M.



Fig. 6. Total spline of the 9-year-old configuration. To transform a Class I geometry to a Class III configuration at this age, vertical stretch is evident with anterior displacement at Ar and anteroposterior stretch along the Go–Id axis. There is some compression of the symphyseal region.



Fig. 7. Total spline of the 11-year-old configuration. To transform a Class I geometry to a Class III configuration, anteroposterior stretch along the Go–Id axis is most evident for this age group. Anterior displacement at Ar and symphyseal deformation are also demonstrable.

In the 11-year-old transformation grid (Fig. 7) from a Class I to a Class III morphology a significant antero-posterior stretch along the Go–Id axis was most evident. Localized vertical compression in the Id–B region was demonstrable but vertical elongation between B and Po was also seen. Horizontal compression was apparent between Gn and M.

In summary, graphical analysis of the Class III mandibular configurations using thin-plate splines demonstrated large spatial-scale deformations affecting the body of a Class I mandible, in combination with localized deformations of its symphyseal region in order to attain a Class III morphology.

#### DISCUSSION

Description of craniofacial shape change from cephalographs is a difficult notion as conventional cephalometry is fundamentally deficient in its application to systematic shape change (Bookstein, 1981). In an attempt to overcome lack of geometric rigour, thin-plate spline analysis was employed on landmark data. This analysis is a function that maps points of a reference configuration on to corresponding points of a test configuration. The thinplate spline program computes and plots the differences between two configurations of landmarks as a transformation grid. Because the only information used to compute a thin-plate spline is the location of landmarks upon an infinitely thin metal plate, and not biological structures, implied homologies of intermediate points must be treated with caution (Rohlf, 1996b). Furthermore, a Class III occlusal type was defined here upon the molar relation (Guyer et al., 1986), even though others have adopted classifications based upon cephalometric analysis (e.g. Enlow et al., 1969), ANB angle (Hashim and Sarhan, 1993), incisor relation (Battagel, 1994) or deciduous canine relation (Tollaro et al., 1996). Despite these limitations, the transformation grids expressing relative displacements of landmarks describing the deformations required to attain a Class III mandibular morphology from a Class I mandibular configuration show good corroboration with conventional investigations that have depended upon conventional cephalometry.

Jacobson (1975) employed the Wits appraisal to demonstrate that in Class III skeletal jaw disharmonies the Wits reading has a negative value. Similarly, Nanda and Merill (1994) used the palatal plane to evaluate sagittal maxillo-mandibular relations. However, these investigators reported that some patients malocclusions were incorrectly diagnosed; the method using the ANB angle and the nasion perpendicular plane (McNamara, 1984) did not indicate any definitive trend. In contrast, Kerr et al. (1992) found that the ANB angle was a potent discriminator when two groups of individuals with severe Class III malocclusion were compared. But Kerr and Ford (1992) assessed the homogeneity of variance in 10-year-old boys and reported that mandibular body length and total mandibular length were the linear dimensions that demonstrated the greatest variability. Therefore, analytical techniques that are not dependent upon reference planes and take account of inequality of variance-covariance matrices are preferable to those that register configurations nominally. In the analysis of the role of mandibular morphology in the aetiology of Class III malocclusions, such techniques may suggest developmental mechanisms that might account for morphological change, unlike those that do not allow for biological size variation and non-Gaussian data distribution.

Hashim and Sarhan (1993) subdivided English children with Class III malocclusions into subgroups according to the value of angle SNB (for protruded mandibles SNB  $\geq 80^{\circ}$ ) and reported significant differences between the groups, suggesting mandibular morphological differences. That these morphological changes arise in the mandibular ramus, corpus and symphyseal regions was demonstrable using thin-plate spline analysis. It was found that articulare was displaced anteriorly in the Class III case. The mandibular condyles of skeletal and dental Class III patients are significantly more anteriorly positioned (Cohlmia et al., 1996). Similarly, Seren et al. (1994) reported relative condylar protrusion associated with anterior mandibular displacement in skeletal Class III malocclusion using axial computerized tomography. Therefore, anterior displacement of articulare is one feature of the Class III mandibular morphology and this appears to be associated with partial warp 4 [Table 1; Fig. 3(c)].

It has been suggested that interdigitation of the posterior teeth may play a part in the regulation of anteroposterior and vertical facial growth (Ostyn et al., 1996). Elimination of interdigitation may result in a more prognathic mandible, as it has been hypothesized that condylar growth is controlled by physical restraint, the absence of which leads to incremental growth (Van Vuuren, 1991). It was found that the mandibular body was subjected to extensive forces acting along the anteroposterior plane associated with its elongation and horizontal stretching. That the deformation of the mandibular corpus is a feature of the Class III mandibular morphology is supported by the large spatial scale of the partial warps [e.g. partial warp 5: Table 1; Fig. 3(b)] that act on this variable. However, when Lu et al. (1993) investigated Japanese adolescents treated for mandibular prognathism with orthopaedic chin-cup therapy, they reported that dimensional changes of the mandible were not easily produced and that postero-inferior repositioning of the mandible played an important part in altering the prognathic profile to a more orthognathic one. Therefore, it appears that mandibular form may be delineated very early on during development and, once ontogenetically established, it is liable to only limited remodelling activities.

The results of mandibular elongation may be more obvious anteriorly in the mental region. Indeed, genioplasty is one procedure that is used in the correction of the mandibular prognathic profile (e.g. Ayoub et al., 1994). We found significant deformation of the mandibular symphyseal region associated with partial warps 1-3 of relatively large magnitudes [Table 1; Fig. 2(d)-(f)]. Therefore, it is possible that anterior displacement of articulare, and elongation of the mandibular corpus allied with symphyseal remodelling, contribute to the appearance of the Class III facial profile with charactermandibular prognathism. istic Despite these findings, Dibbets (1996) asserts that in juveniles the mid-face above anything else creates the characteristic difference between the three Angle classes, not the mandible. While it has been shown here that mandibular morphology may play a part in the determination of facial profile, new findings for midfacial morphometry are reserved for future reference. Further studies based upon finite-element morphometry, including an analysis of the nasomaxillary complex, will enable both localization and quantification of allometry within the mandible, and will delineate a basis for the cellular and molecular analyses required for the study of mid-facial retrusion.

In summary, this study tested the hypothesis that a specific pattern of mandibular morphology is associated with Class III malocclusion, and the deformations involved have been visualized using thinplate spline analysis. Large spatial-scale deformations were found to affect the body of the mandible (partial warp 5) in nearly all age groups tested, whereas localized shearing was more apparent anteriorly (partial warps 1-3). Thus, a combination of large-scale and localized deformations appears to contribute to a mandibular geometry that may reflect the biological mechanism responsible for the differences in craniofacial morphology for Class I and Class III individuals. Sarnat (1986) suggests that, in young mammals, condylar proliferation increases the mandibular corpus between articulare and pogonion. In our study, based upon human cephalographs, both moderate deformations in the condylar region of the ramus and significant alterations affecting the body of the mandible predominantly associated with symphyseal changes, could be identified. That the deformation of the mandibular corpus is a significant component in the aetiology of Class III malocclusions is supported by the magnitudes of the partial warps describing the transformations involved. Our study supports the theory that alteration in mandibular growth patterns leads to shape changes associated with a Class III appearance. Moreover, it is possible that the alteration is likely to include condylar proliferation allied with corpus and symphyseal remodelling.

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